

$$\text{map}(f \circ g) == \text{map}(f) \circ \text{map}(g)$$

$$\begin{array}{l} f :: B \Rightarrow C \\ g :: A \Rightarrow B \end{array} \quad f \circ g :: A \Rightarrow C$$

```
trait CovariantFunction[F[_]] {
  def map[A, B](f: A => B): F[A] => F[B]
}

trait Bar[A] {
  def bar(x: X): A
}

object CoBar extends CovariantFunction[Bar] {
  def map[A, B](f: A => B): Bar[A] => Bar[B] = (barA: Bar[A]) => new Bar[B] {
    def bar(x: X): B = f(barA.bar(x))
  }
}
```

$$\begin{array}{l} \text{barA} :: \text{Bar}[A] \\ x :: X \end{array}$$

$$\begin{array}{l} \text{map}(f \circ g) :: \text{Bar}[A] \Rightarrow \text{Bar}[C] \\ \text{map}(f \circ g)(\text{barA}).\text{bar}(x) = \\ (f \circ g)(\text{barA}.\text{bar}(x)) \end{array}$$

$$\begin{array}{l} \text{map}(f) \circ \text{map}(g) :: \text{Bar}[A] \Rightarrow \text{Bar}[C] \\ (\text{map}(f) \circ \text{map}(g))(\text{barA}).\text{bar}(x) = \\ \text{map}(f)(\text{map}(g)(\text{barA})).\text{bar}(x) = \\ f(\text{map}(g)(\text{barA}).\text{bar}(x)) = \\ f(g(\text{barA}.\text{bar}(x))) = \\ (f \circ g)(\text{barA}.\text{bar}(x)) \end{array}$$

Thus Bar preserves morphism composition.